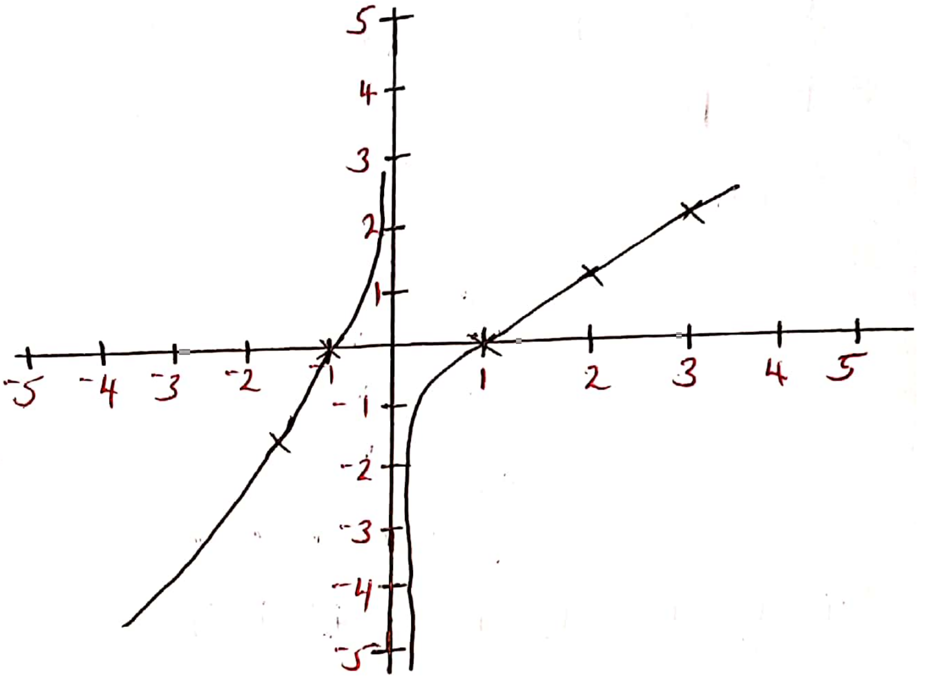
MATHEMATIC ASSIGNMENT

1. $\lim_{x \to -2} \frac{x^2 - 1}{2x}$

(a) Graphing Utility

Table of Values

X	Y
-2	-0.75
-1	0
1	0
2	0.75
3	1.333



(b) Algebraic Solution

$$\lim_{x \to -2} \frac{x^2 - 1}{2x}$$

$$= \cancel{\lim_{x \to -2}} = \frac{(-2)^2 - 1}{2(-2)}$$

$$= \frac{4 - 1}{-4}$$

$$= \frac{-3}{4} \text{ or } -0.75$$

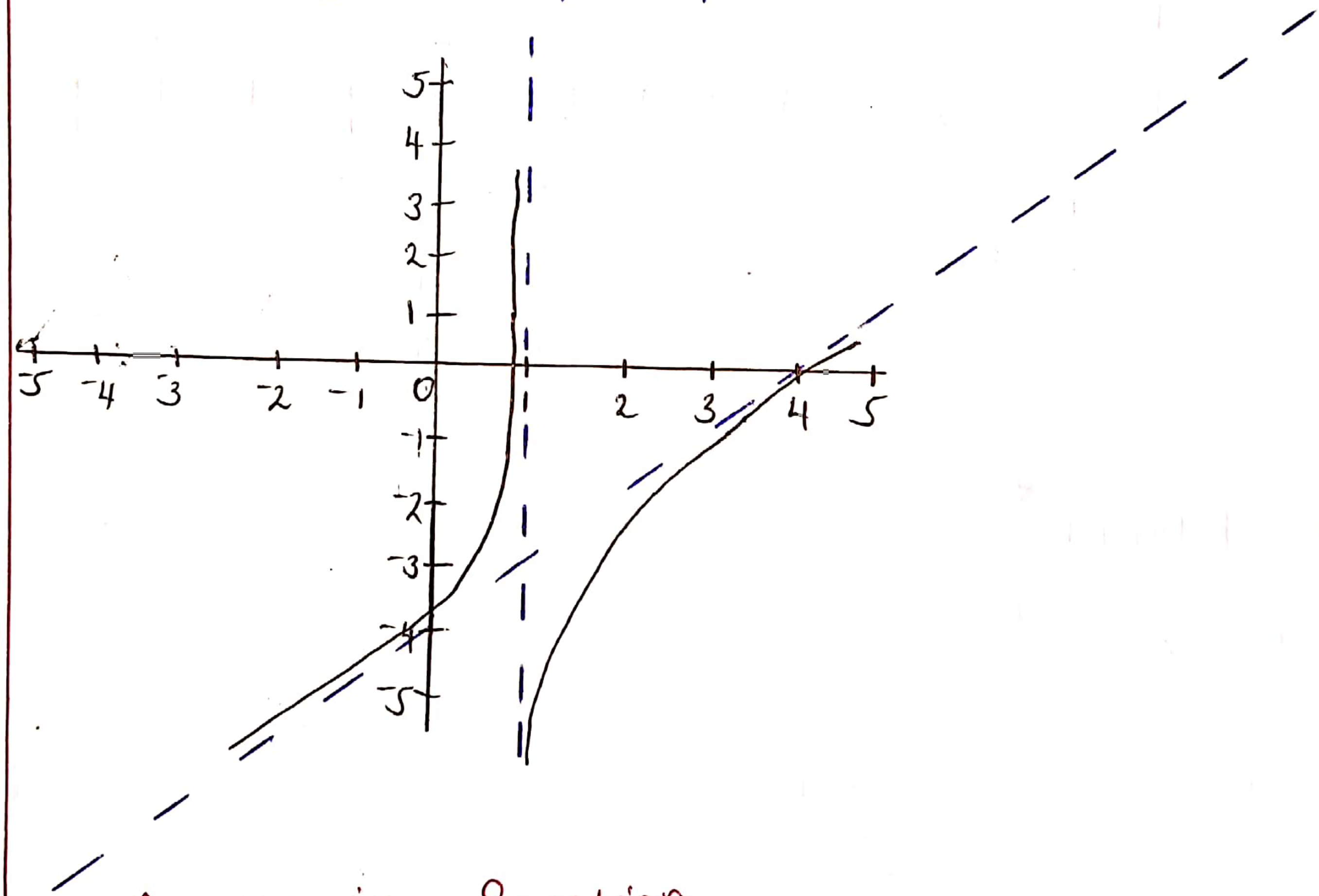
2. $$\lim_{x \to 1} \frac{-x^2+5x-3}{1-x}$$

(a) Graphing Utility

Vertical Asymptotes : $x = 1$

No Horizontal Asymptotes

Oblique Asymptotes: $y = x - 4$



(b) Algebraic Solution

$$\lim_{x \to 1} \frac{-x^2+5x-3}{1-x}$$

$$= \frac{-(1^2)+5(1)-3}{1-1}$$

$$= \frac{-1+5-3}{1-1}$$

$$\frac{1}{0} = \infty$$

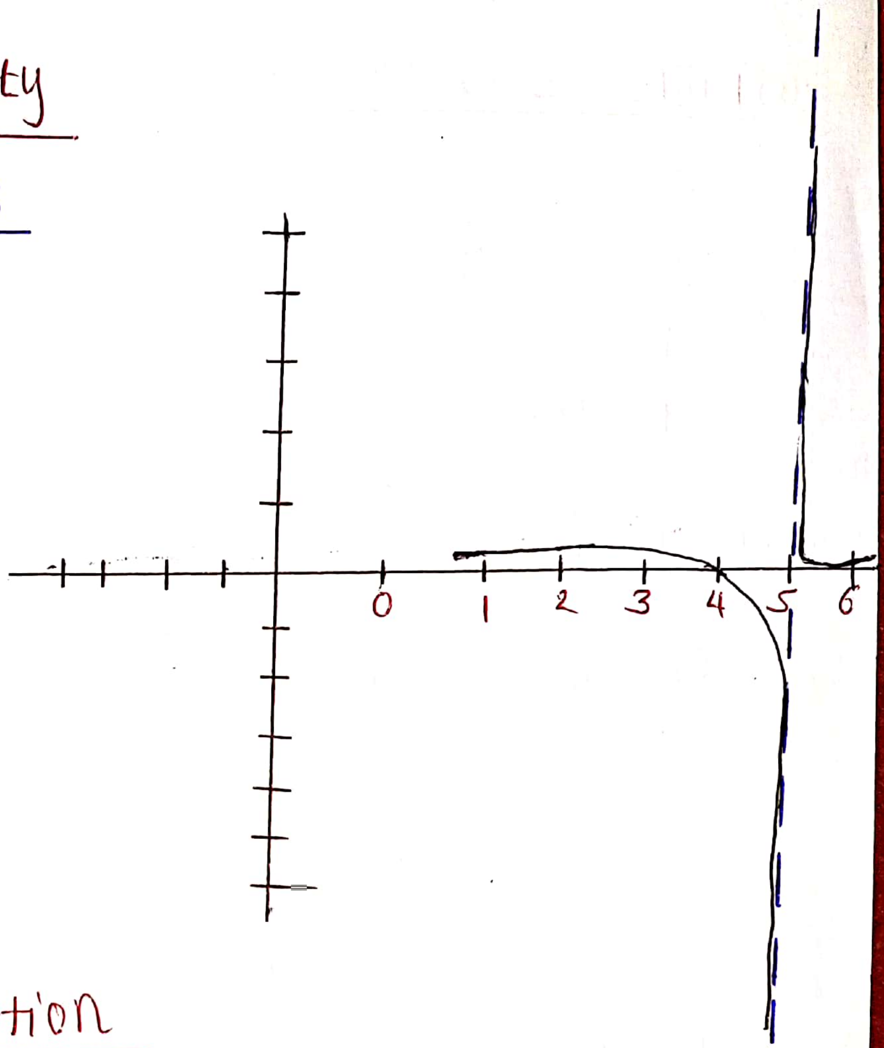
Hence this limit does not exist so it diverges

3. $$\lim_{x \to 5} \frac{\sqrt{x} - 2}{x - 5}$$

(a) Graphing Utility

Table of Values

X	Y
1	0.25
2	0.195
4	0
6	0.449
7	0.323
8	0.276



(b) Algebraic Solution

$$\lim_{x \to 5} \frac{\sqrt{x} - 2}{x - 5}$$

$$\Rightarrow \frac{\sqrt{5} - 2}{5 - 5}$$

$$= \frac{\sqrt{5} - 2}{0} = \infty$$

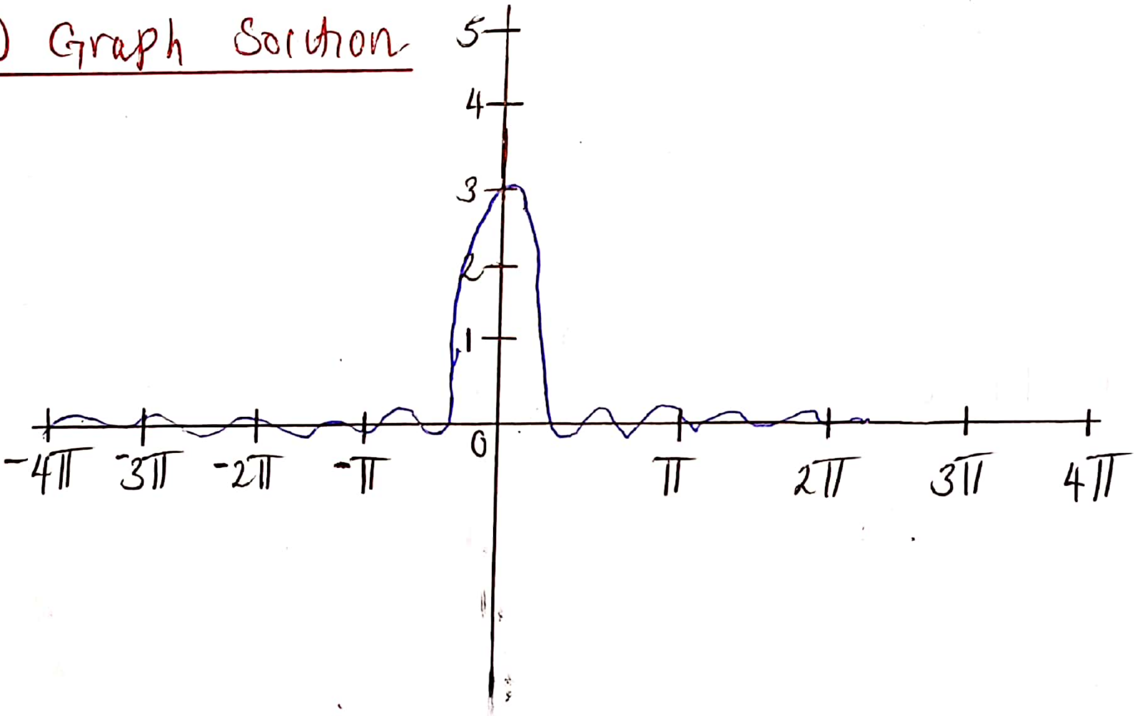
Hence the limit does not exist

4. $\lim\limits_{x \to 0} \dfrac{\sin 3x}{x}$

(a) Table Solution

x	y
-2	0.0522
-1	0.0523
1	0.0523
2	0.0522

(b) Graph Solution



(c) Actual limit

$\lim\limits_{x \to 0} \left(\dfrac{\sin 3x}{x} \right)$

= Applying L Hopital Rule

$\lim\limits_{x \to 0} \left(\dfrac{\cos(3x) \cdot 3}{1} \right)$

Putting 0 where we have X

$= \dfrac{\cos(3 \cdot 0) \cdot 3}{1}$

Simplify

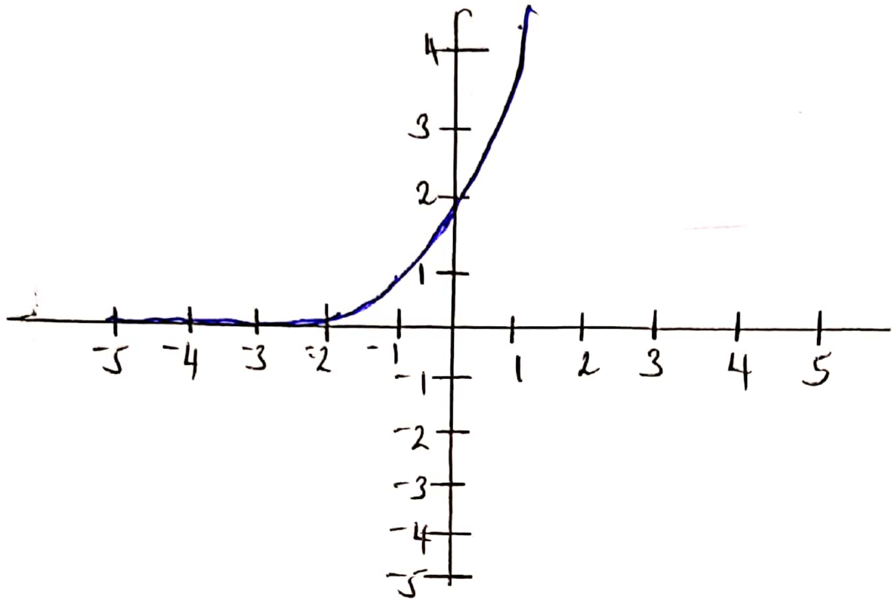
$\dfrac{\cos 0 \times 3}{1}$

$= \dfrac{1 \times 3}{1}$

$= 3$

5. $\lim_{x \to 0} \frac{e^{2x}-1}{x}$

(a) Graphing



(b) Solution

$\lim_{x \to 0} \frac{e^{2x}-1}{x}$

Apply L Hopital's Rule

$= \lim_{x \to 0} \left(\frac{2e^{2x}}{1}\right)$

= Plug in the value of $x = 0$

$\frac{2e^{(2 \times 0)}}{1}$

$= \frac{2 \times 1}{1} = \underline{2}$

6 (a) $f(x) = 3x^2 - 5x - 2, (2,0)$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$= 6x - 5$$

At point $(2,0)$ we have;

$$(6 \times 2) - 5 = \underline{\underline{7}}$$

(b) $f(x) = 2x^3 + 6x, (-1, -8)$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$= 6x^2 + 6$$

At point $(-1, -8)$

$$= 6(-1)^2 + 6$$

$$= (6 \times 1) + 6$$

$$= 6 + 6 = \underline{\underline{12}}$$

7. $f(x) = 5 - \frac{2}{5}x$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$= \underline{\underline{\frac{-2}{5}}}$$

8. $f(x) = 2x^2 + 4x - 1$

$\frac{d}{dx}(2x^2) + d/_{dx}(4x) - d/_{dx}(1)$

$= 4x + 4 + 0$

$= \underline{\underline{4x + 4}}$

9. $f(x) = \frac{1}{x+3}$

Appying exponent rule:

$= \frac{d}{dx}\left(\frac{1}{x+3}\right) = d/_{dx}\left((x+3)\right)^{-1}$

Appying chain rule

$-\frac{1}{(x+3)^2} d/_{dx}(x+3)$

$\frac{d}{dx}(x+3) = 1$

$= -\frac{1}{(x+3)^2} \times 1$

$= \underline{\underline{-\frac{1}{(x+3)^2}}}$

10 $\lim_{x \to \infty} \frac{6}{5x-1}$

$= \cancel{6 \times \lim}$

$6 \left(\lim_{x \to \infty} \frac{1}{5x-1} \right)$

$= 6 \cdot \frac{\lim_{x \to \infty} (1)}{\lim_{x \to \infty} (5x-1)}$

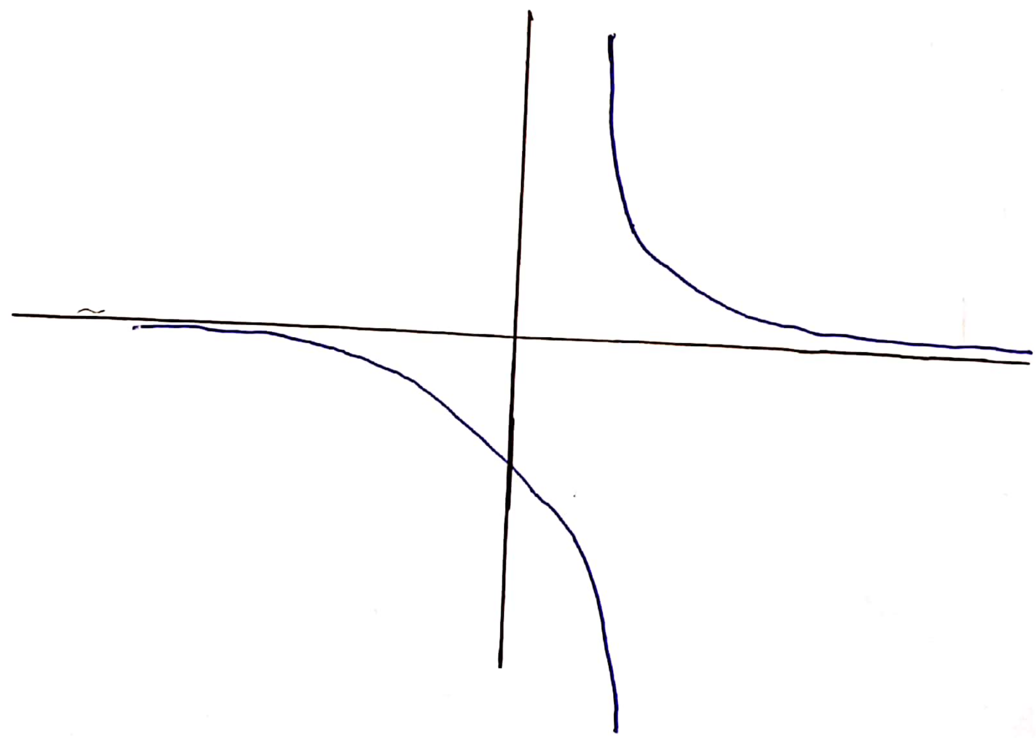
$\Rightarrow \lim_{x \to \infty} (1) = 1$

$\lim_{x \to \infty} (5x-1) = \infty$

$= 6 \times \frac{1}{\infty} = 0$

Hence the limit converge at $\underline{\underline{0}}$

Graphing



11. $$\lim_{x\to\infty} \frac{1-3x^2}{x^2-5}$$

Divide by highest denominator power;

$$\frac{\frac{1}{x^2}-3}{1-\frac{5}{x^2}}$$

$$= \lim_{x\to\infty}\left(\frac{\frac{1}{x^2}-3}{1-\frac{5}{x^2}}\right)$$

$$= \frac{\lim_{x\to\infty}\left(\frac{1}{x^2}-3\right)}{\lim_{x\to\infty}\left(1-\frac{5}{x^2}\right)}$$

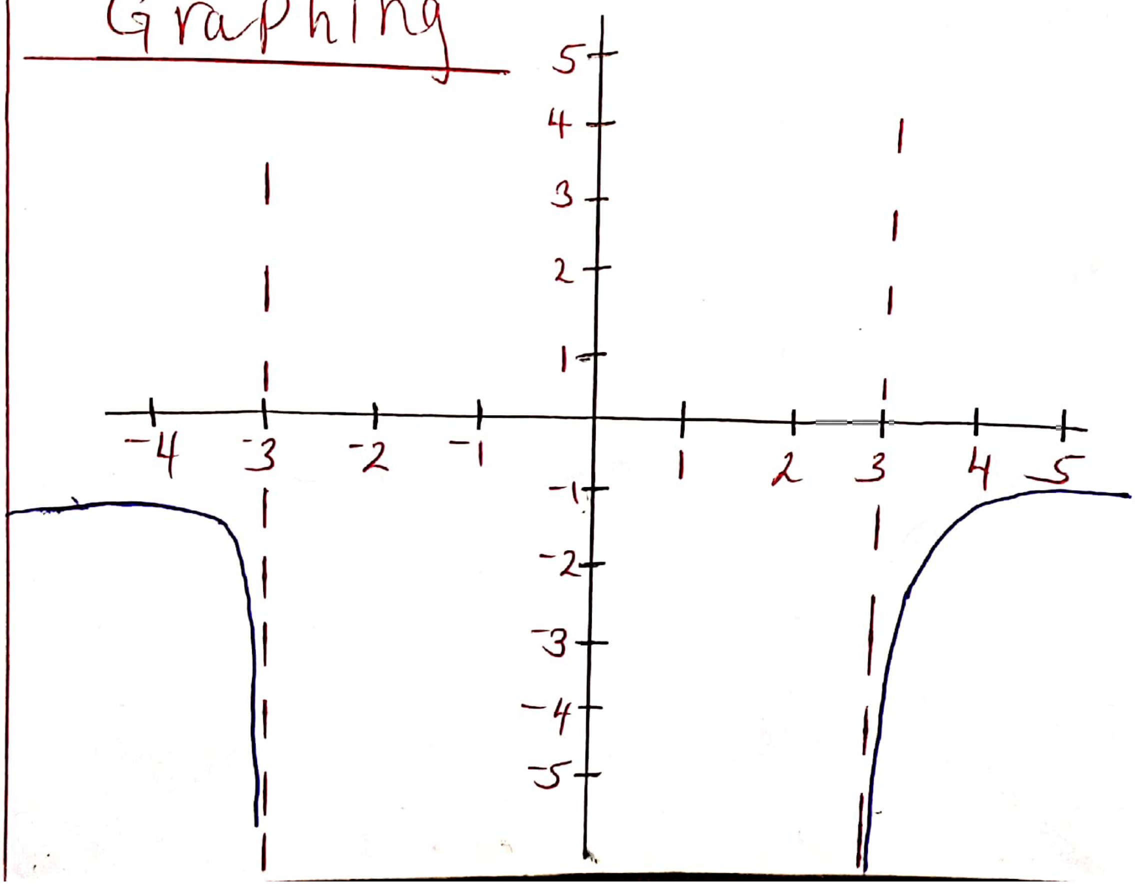
$$\Rightarrow \lim_{x\to\infty}\left(\frac{1}{x^2}-3\right) = \left(\frac{1}{\infty}-3\right) = 0-3$$
$$= -3$$

$$\Rightarrow \lim_{x\to\infty}\left(1-\frac{5}{x^2}\right) = 1-\frac{5}{\infty} = 1-0$$
$$= 1$$

Hence we get $\frac{-3}{1} = -3$

Hence the limit exist at -3

Graphing



12 $\lim\limits_{x \to -\infty} \dfrac{x^2}{3x+2}$

Divide by highest denominator power:

$\dfrac{x}{3+2/x}$

$= \lim\limits_{x \to -\infty} \left(\dfrac{x}{3+\frac{2}{x}}\right)$

$= \dfrac{\lim\limits_{x \to -\infty} (x)}{\lim\limits_{x \to -\infty} \left(3+\frac{2}{x}\right)}$

$\lim\limits_{x \to -\infty} (x) = -\infty$

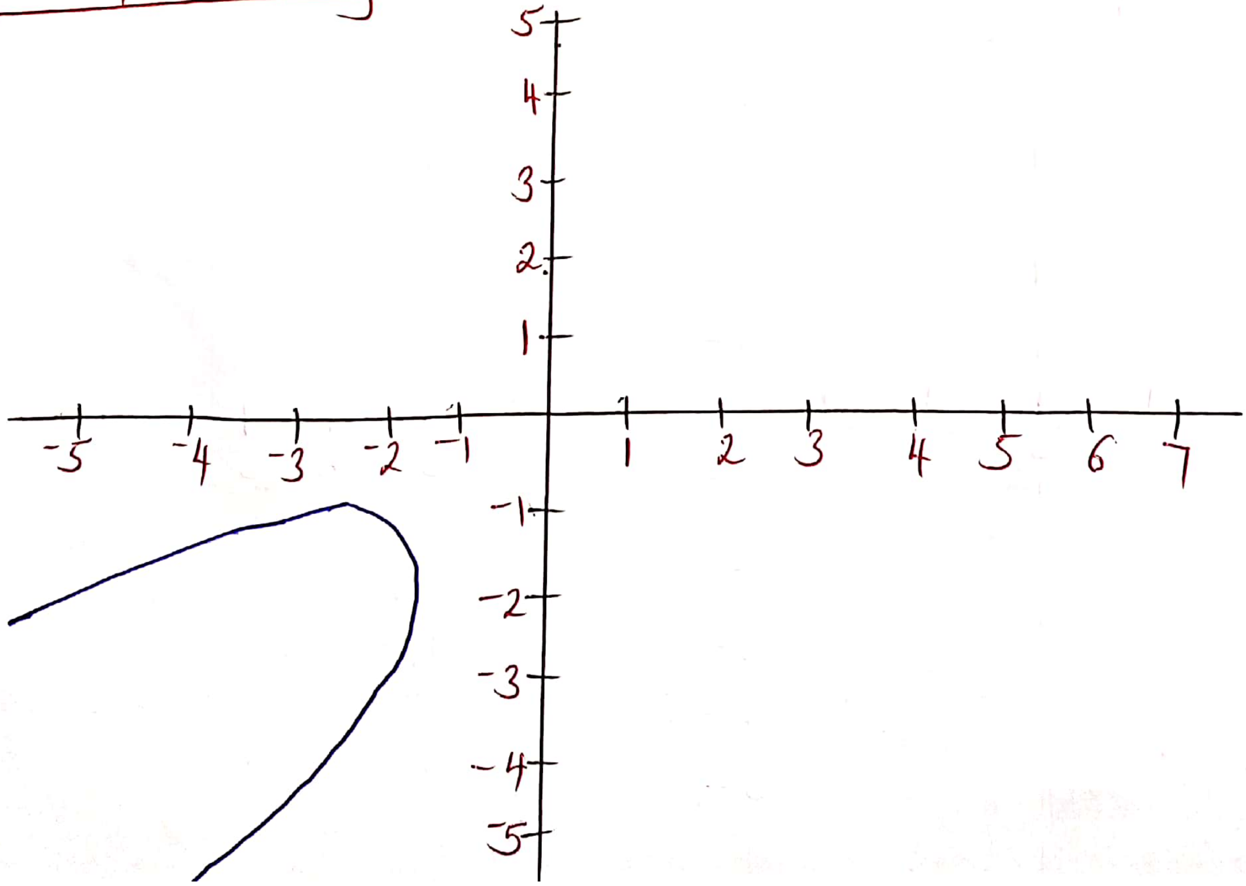
$\lim\limits_{x \to -\infty} \left(3+\frac{2}{x}\right) = 3 + \frac{2}{-\infty} = 3+0$

$= 3$

$= \dfrac{-\infty}{3} = -\infty$

Hence this limit does not converge but rather diverge towards negative infinity and so the limit does not exist.

Graphing



13 $a_n = \dfrac{n^2+3n-4}{2n^2+n-2}$

Divide by highest denominator power:

$$\frac{\frac{n^2}{n^2}+\frac{3n}{n^2}-\frac{4}{n^2}}{\frac{2n^2}{n^2}+\frac{n}{n^2}-\frac{2}{n^2}}$$

$$= \frac{1+\frac{3}{n}-\frac{4}{n^2}}{2+\frac{1}{n}-\frac{2}{n^2}}$$

1st term

$$\frac{1+\frac{3}{1}-\frac{4}{1^2}}{2+\frac{1}{1}-\frac{2}{1^2}} = \underline{\underline{0}}$$

2nd term

$$\frac{1+\frac{3}{2}-\frac{4}{2^2}}{2+\frac{1}{2}-\frac{2}{2^2}} = \underline{\underline{\frac{3}{4}}}$$

3rd term

$$\frac{1+\frac{3}{3}-\frac{4}{3^2}}{2+\frac{1}{3}-\frac{2}{3^2}} = \underline{\underline{\frac{14}{19}}}$$

4th term

$$= \frac{1+\frac{3}{4}-\frac{4}{4^2}}{2+\frac{1}{4}-\frac{2}{4^2}} = \underline{\underline{\frac{12}{17}}}$$

5th term

$$\frac{1+\frac{3}{5}-\frac{4}{5^2}}{2+\frac{1}{5}-\frac{2}{4^2}} = \underline{\underline{\frac{36}{53}}}$$

Limit

$$\lim_{n\to\infty}\left(\frac{1+\frac{3}{n}-\frac{4}{n^2}}{2+\frac{1}{n}-\frac{2}{n^2}}\right)$$

$$= \frac{\lim_{n\to\infty}\left(1+\frac{3}{n}-\frac{4}{n^2}\right)}{\lim_{n\to\infty}\left(2+\frac{1}{n}-\frac{2}{n^2}\right)}$$

$$\lim_{n\to\infty}\left(1+\frac{3}{n}-\frac{4}{n^2}\right)=1$$

$$\lim_{n\to\infty}\left(2+\frac{1}{n}-\frac{2}{n^2}\right)=2$$

Hence $= \underline{\underline{\frac{1}{2}}}$ so the limit exists at $\underline{1/2}$

14. $a_n = \frac{1+(-1)^n}{n}$

1st term

$\frac{1+(-1)^1}{1} = \underline{\underline{0}}$

2nd term

$\frac{1+(-1)^2}{2} = \underline{\underline{1}}$

3rd term

$\frac{1+(-1)^3}{3} = \underline{\underline{0}}$

4th term

$\frac{1+(-1)^4}{4} = \underline{\underline{1/2}}$

5th term

$\frac{1+(-1)^5}{5} = \underline{\underline{0}}$

Limit

$\lim_{n\to\infty} \frac{1+(-1)^n}{n}$

Expand

$\frac{1}{n} + \frac{(-1)^n}{n}$

$= \lim_{n\to\infty} \left(\frac{1}{n} + \frac{(-1)^n}{n}\right)$

$= \lim_{n\to\infty} \left(\frac{1}{n}\right) + \lim_{n\to\infty} \left(\frac{(-1)^n}{n}\right)$

$\Rightarrow \lim_{n\to\infty} \left(\frac{1}{n}\right) = 0$

$\Rightarrow \lim_{n\to\infty} \frac{(-1)^n}{n} = 0$

$0 + 0$

$= \underline{\underline{0}}$

Hence the limit of this seqvence is at $\underline{\underline{0}}$

It exists at 0

15 Given graph is: $f(x) = 8 - 2x^2$

Here, the interval $[0, 2]$ divided into 4 equal parts.

So, $\Delta x = \frac{2-0}{4} = \frac{1}{2} = 0.5$

The end points are: $0, 0.5, 1, 1.5, 2$

Left end points are: $0, 0.5, 1, 1.5$

Now, $f(0) = 8 - 2(0)^2 = 8 - 0 = 8$

$f(0.5) = 8 - 2(0.5)^2 = 7.5$

$f(1) = 8 - 2(1)^2 = 8 - 2 = 6$

$f(1.5) = 8 - 2(1.5)^2 = 8 - 2(2.25) = 3.5$

Therefore required area

$\approx \Delta x \cdot f(0) + \Delta x \cdot f(0.5) + \Delta x \cdot f(1) + \Delta x \cdot f(1.5)$

$\approx \Delta x [f(0) + f(0.5) + f(1) + f(1.5)]$

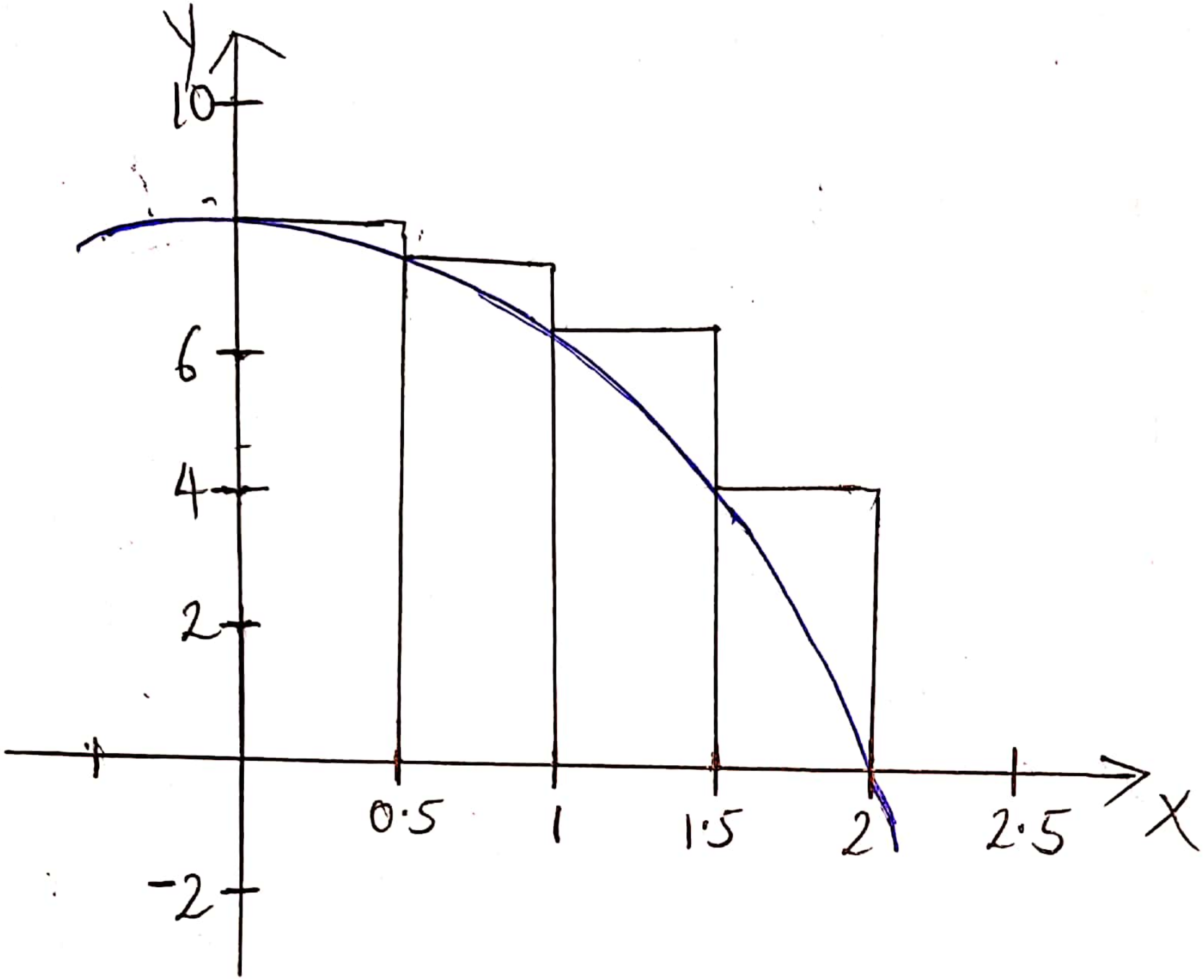
$\approx 0.5 [8 + 7.5 + 6 + 3.5]$

$\approx 0.5(25)$

≈ 12.5

Y

Graphing Utility



16 (b) $f(x) = x+2 : [-2, 2]$

Here, we have to find the area of the region between the graph of function and x-axis using limit process

$$\text{Area, } A = \int_{x=a}^{x=b} f(x)\,dx$$

$$A = \int_{-2}^{2} (x+2)\,dx = \int_{-2}^{2} x\,dx + 2\int_{-2}^{2} dx$$

$$= \frac{1}{2}\left[x^2\right]_{-2}^{2} + 2\left[x\right]_{-2}^{2}$$

$$= \frac{1}{2}\left[4-4\right] + 2\left[2+2\right] = 0 + (2\times 4)$$

$$= 8$$

Therefore $\int_{x=-2}^{x=2} (x+2)\,dx = 8$

17 $f(x) = 3 - x^2 : [-1, 1]$

Area $A = \int_{x=a}^{x=b} f(x)\,dx$

$$A = \int_{-1}^{1} (3 - x^2)\,dx = 3\int_{-1}^{1} dx - \int_{-1}^{1} x^2\,dx$$

$$= 3\left[x\right]_{-1}^{1} - \frac{1}{3}\left[x^3\right]_{-1}^{1}$$

$$= 3[1+1] - \frac{1}{3}[1+1]$$

$$= 6 - \frac{2}{3} = \frac{16}{3}$$

Hence $\int_{x=-1}^{x=1} (3 - x^2)\,dx = \frac{16}{3}$

18

Time (Seconds), x	Height (feet) y
0	0
1	1
2	23
3	60
4	115
5	188

Part a Solution

Here, we use regression feature of a graphing utility to find a quadratic model

$y = ax^2 + bx + c$ for the data given in the table

$x = 0,\ y = 0$

$y = ax^2 + bx + c$

$\Rightarrow 0 = 0 + 0 + c$

$\Rightarrow c = 0$

$y = ax^2 + bx + c$

When $x = 1,\ y = 1$

$\Rightarrow \cancel{1+0}$

$\Rightarrow 1 = a + b + 0$

$a + b = 1 \ldots\ldots\ldots$ (1)

When $x = 2,\ y = 23$

$23 = 4a + 2b + 0$

$\Rightarrow 4a + 2b = 23 \ldots\ldots\ldots$ (2)

$4a + 2b = 23$
$a + b = 1$ $\Rightarrow$ $4a + 2b = 23$
$2a + 2b = 2$

$2a = 21$

$a = \frac{21}{2}$

Hence $b = \frac{-19}{2}$

So $y = \frac{21}{2}x^2 - \frac{19}{2}x$

Part b Solution

$y = \frac{21}{2}x^2 - \frac{19}{2}x$

$v = \frac{dy}{dx}$

$\Rightarrow v = \frac{\partial\left(\frac{21}{2}x^2 - \frac{19}{2}x\right)}{\partial x}$

$v = 21x - \frac{19}{2}$

$v\Big|_{x=5} = (21 \times 5) - \frac{19}{2}$

$v = 105 - \frac{19}{2}$

Hence $v = \frac{191}{2}$ feet/sec

which is the velocity of shuttle after 5 seconds